**Proceedings of the 40th IEEE
Conference on Decision and Control
Orlando, Florida USA, December 2001**

ThP10-3

Feedback Control Systems as Users of a Shared Network: Communication Sequences that Guarantee Stability[1]

Dimitris Hristu-Varsakelis
Department of Mechanical Engineering
University of Maryland, College Park
hristu@glue.umd.edu

Abstract

We investigate the stability of a collection of systems which are governed by linear dynamics and operate under limited communication. We view each system and its feedback controller as users on an idealized shared network which grants access only to a few system-controller pairs at any one time. A communication sequence, which plays the role of a network admission policy, specifies the amount of time available for each system to complete its feedback loop. Using Lyapunov theory, we give a sufficient condition for the existence of a stabilizing communication sequence and show how one can be constructed in a way that minimizes network usage. Our solution depends on the parameters of the underlying system(s) and on the number of controller-plant connections that can be maintained simultaneously. We include simulation results illustrating the main ideas.

1 Introduction

Over the past decade, the rapid growth of communication and network technologies has arguably been characterized by its emphasis on *information flow* (the Internet and mobile telephony being two of the most prominent examples). At the same time, the technologies responsible for what one might call "the connectivity revolution" have significant implications for the design, deployment and control of distributed and networked control systems, fueling recent interest in new large-scale systems such as formations of robots, smart structures and sensor/actuator networks. However, as we might expect and can observe experimentally [6] in such settings, the effectiveness of a control policy depends on the communication constraints imposed by medium which connects sensors, actuators and computing elements within a distributed system. This fact separates networks which mainly transmit information for Internet-based applications, telephony, etc, from those whose "users" are parts of a dynamical system. This important distinction - sometimes summarized in the phrase "networks for control vs. control of networks" - underscores the long-term need for a brand of control theory which balances control and communication considerations.

This paper investigates the stability of a collection of control systems (assumed to be LTI) whose feedback loops are closed via a shared network; the network cannot accommodate all controller-system pairs simultaneously. Because of the communication constraints thus imposed, the systems in question become coupled to one another, so that their stability depends both on the choice of control law(s) and on the allocation of time on the shared network. We want to find: i) communication patterns which allow every system to periodically (but sparsely) close its feedback loop in order to maintain stability and ii) criteria for designing efficient communication patterns which stabilize the entire collection.

A number of related works have explored the effects of communication constraints on control problems, including the relationship between practical stability of a dynamical system and the bit-rate available for feedback [11], control/scheduling co-design [3] and joint communication/control optimization problems [2, 7]. These last works focus on the use of *communication sequences* for quantifying the amount of "attention" received by the sensors and actuators of a control system over the course of a control task. A similar approach has been used to attack problems in optimal tracking [6] and LQG control [9] with limited communication. Recent works which are relevant in the present setting include studies of the effects of network delay on the stability of linear systems [10], algorithms for stabilization of discrete-time systems under limited communication [7], analyses of event-driven asynchronous dynamical systems [5] and switched systems (see [8] and references therein).

2 LTI systems as users of a shared network

Consider a collection of continuous-time LTI systems

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t); \quad i = 1, \ldots, N \quad (1)$$
$$x_i(t) \in \mathbb{R}^n, u_i(t) \in \mathbb{R}^m$$

whose open loop dynamics are unstable. Each system communicates with a remotely located controller that

[1]This work was supported by ARO MURI GC169368NGD

occasionally transmits control signals over a shared network, according to the static state feedback law[1] $u_i(t) = K_i x_i(t)$ (see Fig. 1). We assume that the constant

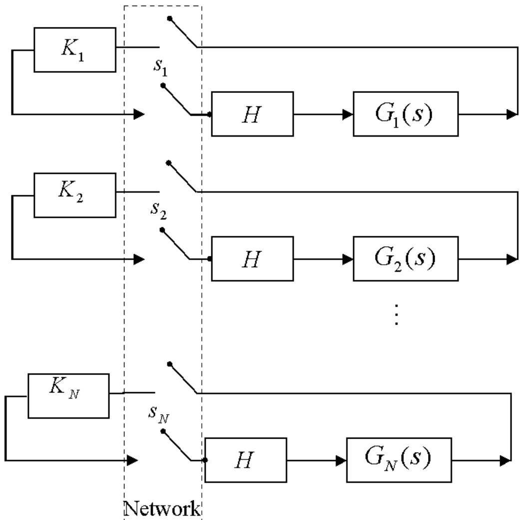

Figure 1: A collection of networked control systems $G_i(s) = I(sI - A_i)^{-1}B_i$ driven by feedback controllers K_i via a network. H denotes a zero-order-hold stage. Only k of N switches s_i can be closed at any one time.

gains K_i are designed to stabilize (A_i, B_i) when the i^{th} feedback loop is closed (i.e $Re\{\lambda(A_i + B_i K_i)\} < 0$, $i = 1, \ldots, N$).

Controller-plant communication is limited in the sense that a maximum of $k < N$ plants may close their feedback loops at any one time. This could be because:

- All systems are stabilized by a single centralized controller which can perform a limited amount of computation per unit time, or has only sufficiently many outputs to communicate with k out of N systems

- Each system has its own controller, with communication taking place over a medium which can only accommodate a maximum of k "users" at a time.

We are interested in *finding an ordered sequence of establishing and terminating communication between each system and its controller (as well as the duration of each communication) in a way that preserves the stability of each system in the collection.* We assume that the "shared network" is an idealized communication medium which provides connectivity between a system and its controller in a discrete sense (*on* or *off*). We will not consider the effects of network layers, random delays or complications due to packed-based communication.

[1] We assume that continuous-time state feedback is available, although the discussion that follows can easily be adapted for output feedback laws $u_i = K_i C_i x_i$ and/or discrete-time dynamics.

For each system we define a binary-valued function $s_i : \mathbb{R}^+ \to \{0, 1\}$ which indicates whether the i^{th} feedback loop is closed at t ($s_i(t) = 1$) or not ($s_i(t) = 0$). Each system is preceded by a zero-order hold stage H which is activated when communication is cut off ($s_i(t) = 0$) and "resets" its output to zero after a certain amount of time (to be specified later) has elapsed without re-establishing communication. The reason for proposing a zero-order hold with such a feature will become clear in the next section. More precisely, let $\mathcal{U}$ denote the space of admissible inputs to one of the systems (e.g. piecewise continuous, $\mathbb{R}^m$-valued functions). We define $H : \mathcal{U} \times \mathbb{R}^+ - \{\infty\} \times \mathbb{R}^+ \to \mathcal{U}$, with

$$H(u(t), t_1, \tau) = \begin{cases} u(t) & t < t_1 \\ u(t_1) & t_1 \leq t < t_1 + \tau \\ 0 & t_1 + \tau \leq t \end{cases} \qquad (2)$$

where t_1 denotes the time that $s_i(t_1^-) = 1, s_i(t_1) = 0$. If communication with the i^{th} system is initiated at $t = t_0$ and interrupted at t_1, we have:

$$u_i(t) = \begin{cases} K_i x_i(t) & t \in [t_0, t_1) \\ K_i x_i(t_1) & t_1 \leq t < t_1 + \tau \\ 0 & t_1 + \tau \leq t \end{cases} \qquad (3)$$

We will call τ_i the "**reset time**" associated with the zero-order hold stage H_i. For notational convenience we will use (A_i, B_i) as shorthand for the LTI system of Eq.1; the triple (A_i, B_i, K_i) will denote the closed-loop dynamics of (A_i, B_i) with static feedback gain K_i. Finally, we will write (A_i, B_i, K_i, τ_i) to indicate the networked feedback system whose zero-order hold stage (defined in Eq.2) has a reset time of τ_i.

2.1 Network allocation and Communication Sequences

In the setting we have just described, the remote controller(s) must choose:

- which of them will communicate with their corresponding system(s) at a particular time
- how long communication should take place before a different set of feedback loops should be closed.

The above remarks motivate the following definition:

Definition 1 *Consider a collection of N LTI systems with limited communication where at most $k < N$ of the systems can simultaneously communicate with their controller(s). A T-periodic* **network allocation sequence** $\gamma_T = \{\Delta s_1, ..., \Delta s_N\}$ *is a choice of N time intervals of length $\Delta s_i > 0$ with $\Delta s_i \leq T$, $\sum_1^N \Delta s_i = kT$.* Δs_i corresponds to the length of time during which the i^{th} system communicates with its controller during every period. Given a T-periodic network allocation sequence, $\gamma_T = \{\Delta s_1, ..., \Delta s_N\}$ we can compute the time intervals during which the i^{th} feedback loop is closed.

$$\mathcal{I}_{ij}(t) = [jT + \sum_0^{i-1} \Delta s_k, jT + \sum_0^{i} \Delta s_k); \quad j = 0, 1, \ldots \qquad (4)$$

Moreover, one can show that there is a corresponding sequence of operations for the switches s_i so that the i^{th} system closes its loop for Δs_i units of time during every interval of duration T, without violating the constraint $\sum_i \Delta s_i < kT$. Equivalently, $Card\{i : t \in \bigcup_j \mathcal{I}_{ij}\} \leq k, \forall t$. We will limit our discussion to periodic communication, although one can consider the problem without this restriction (see for example [5]). Periodic communication ensures that disturbances in a system will not grow too large between communication events. We can now state the problem we are concerned with:

Problem Statement 1 *Consider a collection of LTI systems (A_i, B_i), $i = 1, ..., N$, with a choice of stabilizing feedback laws $u_i = K_i x_i$. If no more than k of those feedback loops can be closed at any time, find a T-periodic network allocation sequence which preserves the stability of the collection.*

A stabilizing network allocation sequence is still stabilizing if we re-order its elements. Once an ordering is chosen, a network allocation sequence defines a set of *continuous-time communication sequence* $\sigma_i(\gamma)$ of times at which the switches $s_i(t)$ transition from 0 to 1 and vise versa[2]. For example, if $k = 1$, $N = 2$ then $\sigma_1(\gamma) = \{0, \Delta s_1, T, T + \Delta s_1, ...\}$ and $\sigma_2(\gamma) = \{\Delta s_1, \Delta s_1 + \Delta s_2, \Delta s_1 + T, \Delta s_1 + \Delta s_2 + T, ...\}$ provided that $\Delta s_1 + \Delta s_2 < T$. Here we use the words "continuous-time" when referring to a communication sequence in order to distinguish it from those in a discrete-time context [2], [7], where communication events occur only at integer multiples of a common period.

3 Sufficient conditions for stability under limited communication

We begin by focusing on a single system-member of the collection[3]. Consider Eq.1 where without loss of generality the system and its controller are allowed to communicate for the first $\Delta s < T$ units of time during each interval $[jT, (j+1)T]$. When access to the network is "allowed" $(s(t) = 1)$ the system can close its feedback loop so that

$$\dot{x} = (A + BK)x; \quad t \in [jT, jT + \Delta s) \quad j = 0, 1, ...$$

Because $(A+BK)$ is stable by choice of K, there exists a quadratic Lyapunov function $V_s(x) = x^T P x$, with $P = P^T > 0$ such that for $Q = Q^T > 0$,

$$(A + BK)^T P + P(A + BK) = -Q < 0 \Rightarrow$$
$$\dot{V}_s(x) \leq \lambda_s V_s(x), \quad t \in [jT, jT + \Delta s), \quad j = 0, 1, ... \quad (5)$$

[2]We note that Prob. 1 can be re-stated in the context of continuous-time communication sequences. Doing so would introduce additional structure and would be more faithful to the operational details of our model. For simplicity, we have chosen to work with network allocation times instead.

[3]To simplify the notation, we will sometimes drop the subscript i when referring to the parameters of a system in the collection

for some $\lambda_s \in \mathbb{R}^-$, with
$0 > \lambda_s \geq 2max_i\{Re(\lambda_i(A + BK))\}$.

We want to estimate the (worst-case) growth rate of $V(x)$ when communication is interrupted $(s(t) = 0)$. Towards that end, let us assume that the zero-order-hold stage preceding the system (A, B) has a reset time of $\tau = 0$. When communication is interrupted (say at $t = t_{int}$) then $H = H(u(t), t_{int}, 0)$, $u = 0$ and the system is governed by its (unstable) open-loop dynamics. We can obtain an upper bound on the growth of $V(x)$ from

$$\dot{x} = Ax; \quad t \in [jT + \Delta s, jT + T) \quad j = 0, 1, ...$$

while at the same time

$$\dot{V}(x) \leq \lambda_u V(x); \quad t \in [jT + \Delta s, jT + T) \quad j = 0, 1, ... \quad (6)$$

for some $\lambda_u \geq 2max_i\{Re(\lambda_i(A))\} > 0$. Notice that if we define $R \triangleq A^T P + PA$ then we have the inequalities

$$0 > \lambda_s \geq max(\lambda(Q))/min(\lambda(P)) \quad (7)$$
$$\lambda_u \geq max(\lambda(R))/min(\lambda(P)) > 0 \quad (8)$$

The bounds for the decay and growth rates λ_s, λ_u suggested by (7),(8) might be conservative, depending on how we choose P. The following result gives a sufficient condition on the amount of time a system must spend periodically closing its feedback loop in order to preserve its stability.

Observation 1 *Consider a networked LTI system described by $(A, B, K, 0)$, with $A + BK$ stable. Let $V(x) = x^T P x, P = P^T > 0$ be a Lyapunov function for the closed-loop system satisfying $(A + BK)^T P + P(A + BK) < \lambda_s P < 0$ when communication is available (feedback loop closed) and $A^T P + PA < \lambda_u P$ otherwise. Then the LTI system is stable under the T-periodic network allocation sequence $\gamma = \{\Delta s\}$ if*

$$\lambda_s \Delta s + \lambda_u (T - \Delta s) < 0, \quad \textit{or equivalently}$$
$$T > \Delta s > \frac{\lambda_u T}{\lambda_u - \lambda_s} \quad (9)$$

Proof: The result follows by considering that for $j = 0, 1, ...$ we have $V(x(jT + \Delta s)) \leq e^{\lambda_s \Delta s} V(x(jT))$ and $V(x(jT + T)) \leq e^{\lambda_u (T - \Delta s)} V(x(jT + \Delta))$ ■

A related result for so-called asynchronous dynamical systems can be found in [5].

3.1 The effect of zero-order-hold $(\tau > 0)$

The bound given by (9) is conservative because it applies even when $\tau = 0$, i.e. the actuators of the linear system are turned off during periods of non-communication. We can obtain a smaller lower bound for Δs_i by examining the case where $\tau > 0$, i.e. a zero-order hold is applied for τ units of time, absent communication.

Theorem 1 *Consider a collection of N networked LTI systems $(A_i, B_i, K_i, \bar{\tau}_i)$, with $(A_i + B_iK_i)$ stable, whose feedback loops can be closed, $k < N$ at a time.*
If $V_i(x) = x^T P_i x$ are Lyapunov functions satisfying $\dot{V}_i < \lambda_{s_i} V_i < 0$ (when the i^{th} feedback loop is closed) and $\dot{V}_i < \lambda_{u_i} V_i$ (i^{th} feedback loop open) then there is a T-periodic allocation sequence
$\gamma_T = \{\Delta s_1, ..., \Delta s_N\}$ which preserves the stability of all N systems if

$$T > \Delta s_i > \frac{\lambda_{u_i}(T - \bar{\tau}_i)}{\lambda_{u_i} - \lambda_{s_i}} \quad (10)$$

$$\sum_{i=1}^{N} \frac{\lambda_{u_i}(T - \bar{\tau}_i)}{\lambda_{u_i} - \lambda_{s_i}} < kT \quad (11)$$

$$\bar{\tau}_i = min_t\{t : max\{\lambda(M_i(t))\} = 0\} \quad (12)$$

where $M_i(t) = (I + \int_0^t e^{-A_i\sigma} d\sigma B_i K_i)^T e^{A_i^T t} P_i e^{A_i t}(I + \int_0^t e^{-A_i\sigma} d\sigma B_i K_i) - P_i$
Proof: Without loss of generality, let $t = 0$ be the time when communication between the i^{th} system and its controller is interrupted. Then, $V_i(0^-) > 0, \dot{V}_i(0^-) < 0$. For $t > 0$ the system evolves according to

$$\dot{x}_i = A_i x_i + B_i K_i x_i(0)$$

or (dropping the subscripts for simplicity)

$$\dot{x} = (A + BK)x - BKe; \quad e(t) \stackrel{\triangle}{=} x(t) - x(0)$$

Because e is continuous on some interval $[0, t')$ (where at t' communication is next restored), $V(x(t))$ is differentiable on $[0, t')$, therefore $\dot{V}$ cannot reverse its sign instantaneously. We conclude that $V(x)$ will continue to decrease for some time before it begins to increase again. In particular, there will be a shortest time $\bar{\tau} > 0$ at which

$$V(x(\bar{\tau})) = x(0)Px(0) \quad (13)$$

i.e. $x^T(0)(I + \int_0^t e^{-A_i\sigma} d\sigma BK)^T e^{A^T t} P e^{At}(I + \int_0^t e^{-A_i\sigma} d\sigma BK)x(0) = x^T(0)Px(0)$
Let $M(t) \stackrel{\triangle}{=} (I + \int_0^t e^{-A_i\sigma} d\sigma BK)^T e^{A^T t} P e^{At}(I + \int_0^t e^{-A_i\sigma} d\sigma BK) - P$, and

$$\bar{\tau} = min_{t,x(0)}\{t : x^T(0)M(t)x(0) = 0\} \Leftrightarrow \quad (14)$$

$$\bar{\tau} = min_t\{t : max\{\lambda(M(t))\} = 0\} \quad (15)$$

if $\tau \leq \bar{\tau}$, then for all $x(0)$, we have $V(x(\tau)) \leq V(x(0))$. Using Obs. 1 we see that if $\bar{\tau}$ is used as a reset time for H, then the evolution of $V(x(t))$ over one period will satisfy $V(x(T)) \leq e^{(T-\bar{\tau}-\Delta_s)\lambda_u} V(x(\Delta_s)) \leq e^{(T-\bar{\tau}-\Delta_s)\lambda_u} e^{\Delta_s \lambda_u} V(x(0))$ from which the sufficient condition for the stability of all N systems follows. ■

Note that we chose to "reset" u to zero after a specified time because holding a value of u for too long might make matters worse for stability, raising $\dot{V}(x)$ over what it would be if $u = 0$. On the other hand, we are guaranteed that if a feedback loop is opened at $t = t_{int}$, $V(x)$ will not grow larger than $V(x(t_{int}))$ for another $\bar{\tau}$ units of time, as long as $u(t) = u(t_{int})$.

3.2 Stability with guaranteed convergence rates

Returning to Th. 1, given $\Delta s_i, \lambda_{s_i}, \lambda_{u_i}$ for each system, the rate of decay of the sequence
$V_i(x_i(jT)),\ j = 1, 2, ..., \quad i = 1, ..., N$ will be

$$r_i \leq \Delta s_i \lambda s_i + (T - \bar{\tau}_i - \Delta s_i)\lambda u_i \quad (16)$$

The rate r_i can take on any value in $[\lambda_{s_i}, (T - \bar{\tau}_i)\lambda_{u_i}]$, where the upper bound of that interval assumes the i^{th} system has a chance to close its loop at least once during $[jT, (j+1)T)$, for an arbitrarily short time. We can now consider a scenario where each system "requests" sufficient network time to achieve a desired rate of convergence, r_i for the samples $V_i(x_i(jT))$. This rate can be viewed as a quantitative measure of the quality of service provided by the network to a dynamical system.

Corollary 1 *Consider a collection of N networked LTI systems $(A_i, B_i, K_i, \bar{\tau}_i)$, with $(A_i + B_iK_i)$ stable, whose feedback loops can be closed $k < N$ at a time. Let $\bar{\tau}_i$ be given by Eq.12. If $V_i(x) = x^T P_i x$ are Lyapunov functions such that $\dot{V}_i < \lambda_{s_i} V_i < 0$ (i^{th} loop closed) and $\dot{V}_i < \lambda_{u_i} V_i > 0$ (i^{th} loop open) then there is a T-periodic allocation sequence $\gamma_T = \{\Delta s_1, ..., \Delta s_N\}$ which preserves the stability of all N systems and guarantees that the sequences $V_i(x_i(jT))$ decay with rates $r_i < 0$ over each period, provided that*

$$q_i \stackrel{\triangle}{=} \frac{\lambda_{u_i}(T - \bar{\tau}_i) - r_i T}{\lambda_{u_i} - \lambda_{s_i}} < T; \quad i = 1, ..., N$$

$$\sum_{i=1}^{N} q_i < kT$$

3.3 Finding a suitable Lyapunov function

The utility of the bounds in Th. 1 and Cor. 1 depends on our estimate of the decay/growth rates $\lambda_{s_i}, \lambda_{u_i}$ for the Lyapunov functions $V_i = x_i^T P_i x_i$. If P_i are not carefully chosen, the inequalities (7),(8) can give conservative results for Δs_i so that a system unnecessarily demands almost constant communication. To avoid this situation, we would like to find quadratic Lyapunov functions for which the the decay/growth rates λ_{s_i},λ_{u_i} are as small as possible (keeping in mind that $\lambda_{s_i} < 0$). The inequalities (10),(11) suggest solving the following problem:

$$\min_P \left(\frac{\lambda_u}{\lambda_u - \lambda_s}\right)$$

subject to

$$(A + BK)^T P + P(A + BK) < \lambda_s P \quad (17)$$

$$A^T P + PA < \lambda_u P \quad (18)$$

$$P > 0, \quad \lambda_s < 0, \quad \lambda_u > 0 \quad (19)$$

Observation 2 *The preceding problem is equivalent to minimizing $c = -\lambda_u / \lambda_s$ subject to (17)-(19).*

Proof: Minimizing $\frac{\lambda_u}{\lambda_u - \lambda_s}$ is equivalent to maximizing the inverse quantity, which is equivalent to minimizing $-\lambda_u/\lambda_s > 0$. ■

We can thus parameterize $\lambda_u = -c\lambda_s$ with $c > 0$ and instead solve the following:

Problem Statement 2 *Given: A, B, K, Minimize $c > 0$ over all $P = P^T > 0$, $\lambda_s < 0$, subject to:*

$$\begin{array}{rcl} (A+BK)^T P + P(A+BK) & < & \lambda_s P \\ A^T P + PA & < & -c\lambda_s P \\ P > 0, \quad \lambda_s < 0, \quad c & > & 0 \end{array} \qquad (20)$$

In this problem we are minimizing a linear function subject to the given inequalities, therefore we expect the minimum to be achieved at the boundary of the feasible region. We observe that the inequalities (20), viewed in the space (P, λ_s, c), are linear in each variable after fixing the other two. Notice that if (P, λ_s, c_1) satisfy the inequalities then so do (P, λ_s, c) for all $c > c_1$. Also, if the inequalities cannot be satisfied for some $c = c_2$, then they cannot be satisfied for any other $c < c_2$. Therefore the minimum in Prob.2 will be the unique value of $c = c^* > 0$ such that (20) has a solution for $c \geq c^*$ but not for $c < c^*$. This observation allows us to find the global minimum c^* using bisection for the parameter c, where at each step we compute a feasible solution to (20) which form a set of bilinear matrix inequalities (in λ_s and P) [1], [4]. The latter problem can be solved globally using a branch and bound technique as suggested in [5].

4 Simulation Results

We simulated a group of three ($N = 3$) unstable linear systems which can communicate two at a time ($k = 2$) with three remotely-located controllers. The communication period was $T = 3sec$. To simplify matters, all three systems are governed by the same dynamics:

$$\dot{x}_i = Ax_i + Bu_i; \quad u_i = Kx_i, \quad i = 1, 2, 3 \qquad (21)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 1.5 & -0.1 \end{bmatrix}; \quad B = [0\ 1]^T,$$

the open loop eigenvalues being $1.17, -1.27$. The controllers $K_i = [-35, -29]$ were designed to place the closed loop eigenvalues of each system $(A_i + B_i K_i)$ at $-1, -2$. We solved the minimization problem of Sec. 3.3 (Prob. 2) and found $\lambda_s = -2.00, \lambda_u = 2.35, c^* = 1.17$ and

$$P_i = P = \begin{bmatrix} 57.1 & 42.5 \\ 42.5 & 35.2 \end{bmatrix},$$

giving a network allocation sequence with $\Delta s_i > 1.62sec$ for each system (i.e. $\gamma_T = \{1.62, 1.62, 1.62\}$). Because $\sum_i \Delta s_i < kT = 6$, there exists a periodic communication sequence that preserves the stability of each of the three systems, closing no more than two of the three feedback loops at any one time. Using the above values for P, A, B, K, we computed (from Eq. 12) $\bar{\tau}_i = 0.72sec$. This suggests that when communication is interrupted (say at $t = t_{int}$) and the zero-order hold is activated, our Lyapunov function has not surpassed its value at t_{int} until another $0.72sec$ have elapsed. From this fact together with Eq.10, we obtained a new bound of $\Delta s_i > 1.23sec$ as a sufficient communication time (every $T = 3sec$) for stabilizing each system. Equivalently, the system(s) can tolerate a communication disruption for at least $1.77sec$, or 59% of the communication period. These numbers are to be compared with those given by the sufficient condition in [10] by which an interruption of less than $t' = 0.173sec$ is required to guarantee stability for the same systems. Figure 2 shows the evolution of the Lyapunov functions $V_i(x_i) = x_i^T P_i x_i^T$ for each of the three systems, each starting from a different, randomly chosen initial condition. The switching functions s_i show-

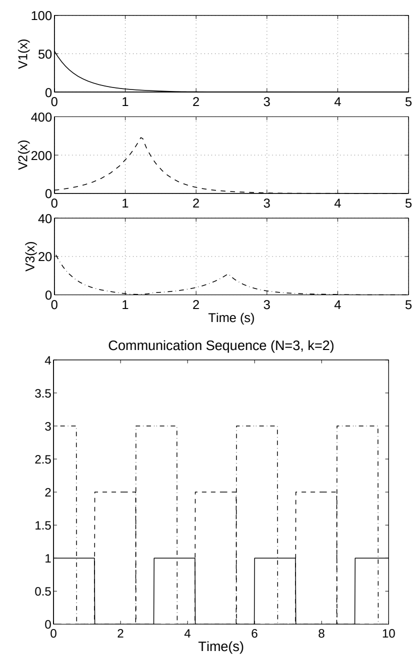

Figure 2: Top: Time evolution of the Lyapunov function of each system. Bottom: The corresponding communication sequence.

ing when communication with the i^{th} system was taking place are plotted in Fig.2, scaled by an integer factor of i to make them easily distinguishable from one another. As expected, there are at most two non-zero indicator functions at any one time. Also, there are time intervals where only one feedback loop is closed. This indicates that there is "room" for guaranteeing a faster rate of convergence.

4.1 Stability with guaranteed convergence rates

Using the same setup as in the previous section, we now ask that the three systems converge with rates

$r_1 = -.5$, $r_2 = -1$, $r_3 = -1.5$, in the sense that $V_i(x_i(jT+T)) \leq e^{r_i} V_i(x(jT))$. Using the results of Cor. 1, we obtained a new network allocation sequence with $\Delta s_1 = 1.57$, $\Delta s_2 = 1.91$, $\Delta s_3 = 2.26$. In this case, $\sum \Delta s_i = 5.74$ which is closer to the limit of available network time. If we did not choose to take into account the effects of the zero-order hold stage (i.e. $\tau_i = 0$) then the worst-case analysis gives $\Delta s_1 = 1.97, \Delta s_2 = 2.31, \Delta s_3 = 2.66$ which sum to $6.94 > kT = 6$ and our criterion would fail to indicate that there exists a communication sequence that stabilizes all 3 systems. Figure 3 shows the evolution of the Lyapunov function for each of the three systems, starting from the same initial conditions used in the previous simulation. The

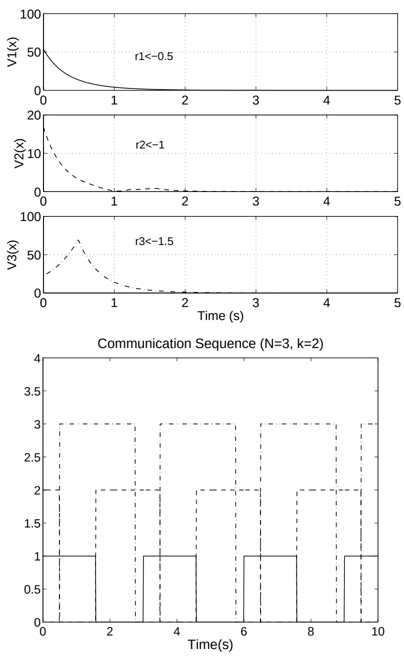

Figure 3: Systems 2 and 3 have faster guaranteed convergence rates resulting from the allocation of more network time to them.

time plot of the communication sequence $i \cdot s_i(t)$ (again scaling s_i by i) shows significantly higher utilization of the available time. Note that the bound of (10),(11) is conservative, in the sense that the actual decay rates observed are significantly faster than those guaranteed.

5 Conclusions and Future Work

We have proposed a model for groups of dynamical systems whose feedback loops are closed via a shared network, and explored the problem of finding network allocation sequences (and their corresponding communication sequences) that preserve the stability of all systems on that network. The presence of communication constraints was modeled by assuming that the network can accommodate only a few controller/plant communications at any one time. Each system's feedback controller was designed without considering the effects of the network. We gave a sufficient condition for the existence of network allocation sequences that preserve the stability of the collection and showed how such sequences can be chosen to minimize the amount of communication necessary for stability. Our analysis has taken into account the effects of zero-order hold applied at the input of each system when communication is not possible. Although our results are conservative, they represent a significant improvement over previous estimates for the amount of communication required to guarantee stability.

Opportunities for further work include the joint selection of the communication sequence and feedback gains, use of time-varying reset intervals in a system's zero-order hold stage, and augmenting the basic model to include dynamic interactions between systems in addition to the coupling introduced by the presence of communication constraints.

References

[1] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*, vol 15 of *Studies in Applied Mathematics*. SIAM, Philadelphia, PA, 1994.

[2] R. W. Brockett. Stabilization of motor networks. In *Proc.* 34^{th} *IEEE CDC*, pg. 1484–8, 1995.

[3] A. Cervin and J. Eker. Feedback scheduling of control tasks. In *Proc.* 39^{th} *IEEE CDC*, pg. 4871–6, 2000.

[4] P. Gahinet and A. Nemirovskii. *LMI Lab: A package for Manipulating and Solving LMIs.* INRIA, 1993.

[5] A. Hassibi, S. P. Boyd, and J. P. How. Control of asynchronous dynamical systems with rate constraints on events. In *Proc.* 38^{th} *IEEE CDC*, pg. 1345–1351, 1999.

[6] D. Hristu. Generalized inverses for finite-horizon tracking. In *Proc.* 38^{th} *IEEE CDC*, vol 2, pg. 1397–402, 1999.

[7] D. Hristu and K. Morgansen. Limited communication control. *Systems and Control Letters*, 37(4):193–205, July 1999.

[8] D. Liberzon and A. S. Morse. Basic problems in stability and design of switched systems. *IEEE Control Systems Magazine*, 19(5):59–70, Oct. 1999.

[9] H. Rehbinder and M. Sanfridson. Scheduling of a limited communication channel for optimal control. In *Proc.* 39^{th} *IEEE CDC*, pg. 1011–16, 2000.

[10] G. Walsh, H. Ye, and L. Bushnell. Stability analysis of networked control systems. In *Proc. American Control Conf.*, pg. 2876–80, 1999.

[11] W. S. Wong and R. W. Brockett. Systems with finite bandwidth constraints - part II: Stabilization with limited information feedback. *IEEE Trans. Automatic Control*, 42(5):1049–1052, 1999.