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LQG control of networked control systems with access constraints and delays

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We explore the LQG control of a networked control system (NCS) in which a linear plant is controlled remotely over a network or other shared communication medium. The medium provides a limited number of simultaneous connections, so that only a subset of the plant's sensors and actuators may communicate with the controller at any one time, subject to known transmission delays. Instead of insisting on jointly optimal control and medium access policies, we reduce the infinity of possible access sequences down to those which preserve the stabilisability and detectability of the underlying plant, and are periodic. Our choice of communication and NCS model effect a kind of 'decoupling' of the LQG problem, in the sense that the medium access policy can be selected independently of the controller. This guarantees the existence of a stabilising LQG controller which is optimal for the communication policy of choice, and which is then combined with a delay compensator. We include simulations that illustrate our approach.

Keywords: networked control systems; medium access constraints; limited communication control; LQG control

1. Introduction

The class of control systems whose feedback loops are closed via networks or other shared communication media has enjoyed steadily-rising interest among researchers during the last decade. In part because of their flexibility, these so-called networked control systems (NCSs) are now encountered in a wide range of domains, from academic laboratories, to commercial vehicles and aerospace applications (Hristu-Varsakelis and Levine 2005). This expansion has been accompanied by several productive efforts to elucidate systems theory at the intersection of control and communication, solve control design problems in the new setting, and quantify the performance of existing methods in the presence of limited communication.

Broadly speaking, the majority of NCSs research has focused on three principal categories of communication constraints which are briefly described next, with discussions typically addressing the effects of a single type of constraint and ignoring the others; see Hristu-Varsakelis (2005) for a recent review. The first category includes NCSs whose communication medium has limited throughput (e.g., bits per second), and one seeks conditions that guarantee various control objectives, such as containability (Wong and Brockett 1997, 1999), stability, and estimation (Nair and Evans 2000; Tatikonda and Mitter 2004). The second includes so-called access constraints, where the shared medium limits the number of sensors and actuators that may communicate with the controller simultaneously. In that setting, NCS stabilisation (Brockett 1995; Hristu and Morgansen 1999; Hristu-Varsakelis and Kumar 2002) and optimal control problems (Rehbinder and Sanfridson 2000; Lincoln and Bernhardsson 2000a) have received perhaps the greatest share of attention. Other work (Azimi-Sadjadi 2003; Imer *et al.* 2004) has examined the question of stability when data transmissions between controller and plant occasionally fail to reach their destination (and are thus referred to as 'dropped packets'). More recently, (Imer and Basar 2006) approached the fundamental question of how much of the network's time should be devoted to measuring versus controlling the underlying plant. Finally, a third class of communication constraints involves transmission delays which may be imposed by the shared medium. Aside from the voluminous literature on time-delay systems, much of the

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analysis there has assumed that the controller was designed *a priori*, and investigated the effect of delays on the stability of the resulting closed loop (e.g., Walsh et al. (2001) and Branicky et al. (2000); see also Zhang et al. (2001) for an informative review). The work of Nilsson (1998) stands out in that area, having explored (along with later extensions (Lincoln and Bernhardsson 2000b; Kao and Lincoln 2004) a substantial array of questions, including stability and optimal control for NCSs that are subject to random delays.

This paper discusses the LQG control of stochastic, linear NCSs, where medium access constraints and delays may be present simultaneously, and one is called to specify the controller and the communication policy that will govern plant/controller interactions. In the present context, the term 'communication policy' is to be understood as a sequence which prescribes the times at which the plant's sensors and actuators are to be granted medium access in order to exchange data with the controller. Arriving at a satisfactory solution of the LQG problem requires coming to terms with the 'strong' coupling between control and communication that frequently arises in NCSs with access constraints. More precisely, the choice of communication policy influences the performance of the controller, just like the choice of controller may result in good performance for some communication policies, but not for others. This leads to technical difficulties and high complexity, if one insists on a jointly optimal solution (see, for example, the LQ problems in Hristu (1999), Rehbinder and Sanfridson (2000) and Lincoln and Bernhardsson (2000a)). Consequently, one must often make strong assumptions regarding the underlying plant (Branicky et al. 2002, Rehbinder and Sanfridson 2000) such as 'block-diagonal' dynamics (Hristu-Varsakelis and Kumar 2002), or medium access constraints that exist for input or output signals, but not for both (Montestruque and Antsaklis 2004, 2005).

The complexity resulting from the interplay between control and communication decisions is sometimes unavoidable; this is especially true in control problems involving NCS with medium access constraints where we insist on optimising simultaneously with respect to the medium access policy as well as the controller. The difficulty of selecting a suitable control-communication policy pair can also depend on the protocol used by the controller and plant, as well as on the NCS model adopted at the outset. For example, depending on the approach taken, the question of whether a stabilising feedback controller exists could be NP-hard – even if the communication policy is fixed in advance e.g., Hristu-Varsakelis and Morgansen (1999) and references therein – or much simpler, Zhang and Hristu-Varsakelis (2006). In particular, the use of a zero order hold (ZOH) at the 'receiving end' of a communication medium (i.e., at the plant's and controller's input stages) may have the effect of introducing time-varying delays and may necessitate state augmentation; see, for example, the 'extensive form' in Hristu-Varsakelis and Morgansen (1999).

The contribution of this paper is to propose a solution to the LQG problem for NCSs by pursuing a 'separation' of the control and communication sub-problems. We explore the use of an NCS architecture whereby the plant and controller forgo the use of a ZOH and instead choose to 'ignore' (in a manner to be made precise) the actuators and sensors that are not actively communicating. This is not essential, but it simplifies the notation and gives a low complexity model. Under the proposed approach, the communication sequences that determine controller-plant interactions can be designed easily and independently from the controller. Furthermore, by proper choice of communication, the problems of state estimation and LQ optimal control for a NCS become a standard LQG problem which can be addressed using existing design tools from LTV systems theory. The separation discussed above is rather fortunate in our case; it does not usually occur in problems involving control of NCSs, and is partly the result of a trade-off. Specifically, we will not attempt to solve the joint problem (i.e., find the optimal communication policy), but instead will identify classes of communication sequences which are 'good enough', in the sense that they make the design of the accompanying LQG controller straightforward. Our approach is motivated by recent results (Zhang and Hristu-Varsakelis 2005b, 2006) on the NCS feedback stabilisation problem; it avoids the complexity associated with previously proposed models and addresses multi-input multi-output (MIMO) NCSs whose dynamics are 'fully coupled'. To the authors' knowledge, this work is the first to provide a solution to the LQG problem under medium access constraints and delays. A preliminary version of this work appeared in Zhang and Hristu-Varsakelis (2005a) and Hristu-Varsakelis (2006). For related work on state estimation for simple NCS configurations see Micheli and Jordan (2002), Sinopoli et al. (2003) and Liu and Goldsmith (2004).

The remainder of this paper is structured as follows. In § 2 we show how a MIMO NCS with medium access constraints can be modelled as a time varying system with a reduced number of inputs and outputs, and pose the LQG problem for NCSs. In § 3 we explore the problem of choosing the communication policies that will manage data exchanges between controller and plant. We prove that for a stabilisable (detectable) plant, it is always possible to design a policy which preserves stabilisability (detectability) after medium access constraints and delays are put in place. Agreeing to always use such a policy will effectively decouple the selection of the controller from that of the communication. § 4 discusses sufficient conditions for the convergence of the Kalman filter and optimal LQ gains associated with the LQG controller. § 5 describes a delay compensation method, inspired by Luck and Ray (1990, 1994), that may be applied when controller-plant communications are subject to known delays. A numerical example is given in § 6.

2. NCS model and problem formulation

Our NCS model is a stochastic version of that in Zhang and Hristu-Varsakelis (2006). Consider an NCS in which a remote controller interacts with a stochastic linear time-invariant (LTI) plant via a shared medium (see Figure 1). We will take $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^p$ to be the plant's state, input, and output vectors, respectively. The medium limits communication in two ways: (i) it imposes communication delays, and (ii) it does not allow simultaneous communication with all of the plant's sensors and actuators. More specifically:

- There are w_σ available *output channels* connecting the sensors to the controller, where $1 \le w_\sigma < p$. That is, only w_σ of the p sensors can transmit their output to the controller at any one time, while others must wait. Furthermore, data transmitted by the plant at time t, arrives at the controller at $t+\tau_{pc}$ for some $\tau_{pc} > 0$.
- Similar constraints apply at the plant's input stage, where m actuators share w_ρ *input channels*, $(1 \le w_\rho < m)$ to receive control signals from the controller. At most w_ρ of the m actuators can access the input channels simultaneously, while controller-to-plant communication is subject to a delay of $\tau_{cp} > 0$.

The delays τ_{cp} and τ_{pc} will be taken to be known. We will ignore any bit-rate constraints or quantisation effects associated with the transmission of data through the communication medium.

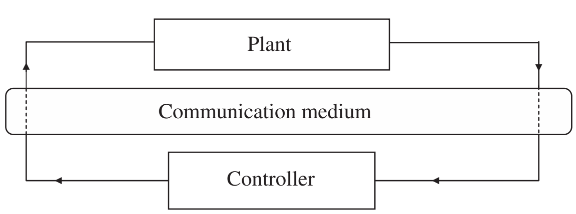

Figure 1. A networked control system with its three principle components.

Based on these assumptions, the NCS takes on the configuration shown in Figure 2, where the 'open' or 'closed' status of the switches indicates the medium access status of the corresponding sensors or actuators. The vectors $\bar{y} \in \mathbb{R}^{w_\sigma}$ and $\bar{u} \in \mathbb{R}^{w_\rho}$ are the controller input and output, respectively, and will generally differ from the plant output y and input u (in a manner which will be made precise shortly) because of the communication constraints described above. We have in mind that the plant may evolve either in discrete or in continuous time (in which case it is sampled periodically). Although our approach will apply to both settings, it will be convenient to begin with the discrete-time case

$$\left.\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + v(k) \\ y(k) &= Cx(k) + w(k), \quad k = 0, 1, 2, \ldots \end{aligned}\right\} \tag{1}$$

The implications of a continuous-time plant will be discussed in § 5. In (1), the disturbances, $v(\cdot)$, $w(\cdot)$, are both Gaussian, i.i.d., with $v(\cdot) \sim N(0, G)$ and $w(\cdot) \sim N(0, I_{p\times p})$, where $I_{p\times p}$ is the $p \times p$ identity matrix and $G = G^T > 0$ is $n \times n$. The initial condition $x(0)$ is assumed to be Gaussian as well, with $x(0) \sim N(x_0, \Sigma_0)$, $\Sigma_0 = \Sigma_0^T > 0$.

The problem we are concerned with is the following.

Problem 1: Given a linear stochastic NCS where the plant (1) communicates with its controller subject to given access constraints (w_ρ, w_σ) and delays (τ_{pc}, τ_{cp}), find a medium access policy for the plant's sensors and actuators, and a control policy, such that the cost function

$$J = \mathcal{E}\left\{\sum_{k=0}^{N_f} x^T(k)Qx(k) + u^T(k)u(k)\right\} \tag{2}$$

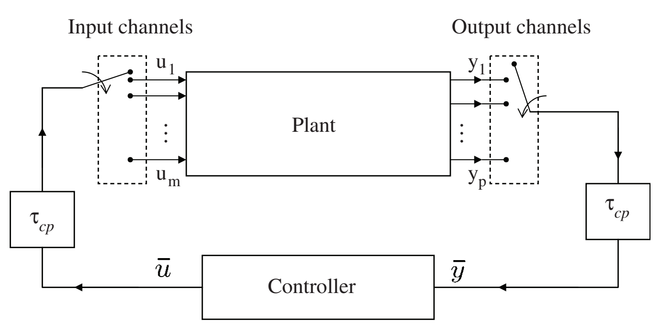

Figure 2. Modelling medium access constraints and transmission delays. The possible positions of the switches indicate which sensor(s)/actuator(s) are granted access to communicate with the controller.

is minimised, where $N_f > 0$, and $\mathcal{E}$ denotes expected value.

We note that in Problem 1 the communication constraints enter indirectly, via the relationship between the transmitted and received information that is exchanged between plant and controller ($\bar{u}$ and u, $\bar{y}$ and y). To describe that relationship we will, for the moment, consider the NCS dynamics under medium access constraints only, assuming a deterministic plant with no transmission delays, i.e.,

$$\left.\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k). \end{aligned}\right\} \tag{3}$$

We will return to the stochastic setting (1) in § 4; in § 5 we will present the required modifications for the case where access constraints and delays are present simultaneously.

2.1 *Communication sequences: modelling access constraints*

For $i = 1, \ldots, p$, let the binary-valued function $\sigma_i(k)$ denote the medium access status of the i-th output, y_i, at time k, i.e., $\sigma_i(k) : \mathbb{Z} \mapsto \{0, 1\}$, where 1 means 'accessing' and 0 means 'not accessing'. The medium access status of all p outputs will be represented by a 'p-to-w_σ communication sequence' (Hristu-Varsakelis and Morgansen 1999; Zhang and Hristu-Varsakelis 2005b),

$$\sigma(k) = [\sigma_1(k), \ldots, \sigma_p(k)]^T.$$

Definition 1: Let $M, N \in \mathbb{N}$ with $N \le M$. An M-to-N communication sequence is a map, $\sigma(k) : \mathbb{Z} \mapsto \{0, 1\}^M$, satisfying $\|\sigma(k)\|^2 = N$, $\forall k$.

As we have previously indicated, at each time k, the controller is to compute plant inputs based only on data from the w_σ sensors which were granted medium access; all others will be effectively ignored. Let the output received by the controller at time k be denoted by $\bar{y}(k) = [\bar{y}_1(k), \ldots, \bar{y}_{w_\sigma}(k)]^T$, i.e., $\bar{y}(k)$ contains those elements from $y(k)$ for which $\sigma_i(k) = 1$. To establish the relationship between $y(k)$ and $\bar{y}(k)$, we will make use of the following notation.

Definition 2: Let $\eta(k)$ be an M-to-N communication sequence. Then, for all $k \in \mathbb{N}$, the $N \times M$ matrix $\mu_\eta(k)$ is obtained by deleting the $M - N$ all-zero rows from the $M \times M$ matrix diag$(\eta(k))$. We will refer to $\mu_\eta(\cdot)$ as the matrix form of η.

Example 1: If $\eta(1) = [1, 1, 0, 1]^T$, then

$$\mu_\eta(1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Using the last definition, we can express $\bar{y}(k)$ as

$$\bar{y}(k) = \mu_\sigma(k) y(k), \tag{4}$$

where $\sigma(k)$ is the output communication sequence. Similarly, the medium access status of the plant's m inputs will be represented by an m-to-w_ρ communication sequence $\rho(k)$. When an input, u_j, loses its access to the communication medium, the plant will ignore that input until the corresponding actuator regains medium access, by setting $u_j = 0$ while $\rho_j = 0$. Let $\bar{u}(k) = [\bar{u}_1(k), \ldots, \bar{u}_{w_\rho}(k)]^T$ denote the elements of $u(k)$ whose actuators were granted medium access and received updated inputs from the controller at time k. Under the protocol outlined above,

$$u(k) = \mu_\rho(k)^T \bar{u}(k). \tag{5}$$

In the sequel, we will refer to $\rho(\cdot)$ and $\sigma(\cdot)$ as the input and output communication sequences, respectively.

By combining (3)–(5), we obtain a linear time-varying (LTV) system with w_ρ inputs and w_σ outputs:

$$\left.\begin{aligned} x(k+1) &= Ax(k) + B\mu_\rho(k)^T \bar{u}(k) \\ \bar{y}(k) &= \mu_\sigma(k) C x(k). \end{aligned}\right\} \tag{6}$$

These equations describe the NCS 'from the controller's point of view'. We will refer to (6) as the *extended plant*; it incorporates the dynamics of the plant together with the access status of the communication medium.

Remark 1: The choice of 'removing' the ZOH elements commonly included in previous NCS models has the effect of avoiding any 'enlargement' of the state vector (to include past states or inputs) and leaves us with an LTV system whose parameters are functions of the input and output communication sequences. This situation is to be contrasted with Brockett (1995) and Hristu-Varsakelis and Morgansen (1999), among others. We also note that the choice of representation for the signals $\bar{u}(k)$ and $\bar{y}(k)$, as well as for the sequences $\mu_\rho(k)$, $\mu_\sigma(k)$ differs from that in Zhang and Hristu-Varsakelis (2005b, 2006), where u and $\bar{u}$ (resp. y and $\bar{y}$) have the same dimensions. Here, we have 'removed' the unavailable outputs and inputs from the NCS model, as seen by the controller whose dimensions are thus reduced ($w_\rho \times w_\sigma$ instead of $m \times p$). In §4.1, 4.2, this choice will help avoid singularities that would arise when solving the matrix Riccati equations associated with the LQG problem. A similar technique is mentioned in Bittanti *et al.* (1991) for multi-rate systems.

Remark 2: Our communication protocol requires that actuators, in a sense, 'turn off' when they are not communicating. Alternatively, it is possible to amend the extended plant model to include zero-order holding of inputs, by enlarging the state to include those inputs which are not updated, i.e.,

$$\left.\begin{aligned} x(k+1) &= Ax(k) + B(I - M_\rho(k))u_{ZOH}(k) \\ &\quad + B\mu_\rho^T(k)\bar{u}(k), \\ u_{ZOH}(k+1) &= (I - M_\rho(k))u_{ZOH}(k) + \mu_\rho^T(k)\bar{u}(k). \end{aligned}\right\} \tag{7}$$

where $M_\rho(k) \triangleq \text{diag}(\rho(k))$, as in Zhang and Hristu-Varsakelis (2006). The discussion that follows can be modified accordingly if one wishes to adopt (7). The details of that approach are straightforward but tedious, and will not be pursued here. See, however, Ionete and Cela (2006).

3. Choosing effective communication sequences

In Problem 1, we are called to decide: (i) the communication sequence which will control traffic on the shared medium, and (ii) the controller which will produce $\bar{u}(\cdot)$. As in most problems involving optimal control of NCS with access constraints, the optimum depends on the choice of communication sequence, as it is the latter that determines the time-varying dynamics of the extended plant (6). Solving the joint problem, i.e., optimising with respect to both control and communication is generally difficult (e.g., Hristu (1999), Rehbinder and Sanfridson (2000) and Lincoln and Bernhardsson (2000a)), and often involves combinatorial complexity. Instead of insisting on the joint optimum, we will solve a relaxed version of Problem 1 using Zhang and Hristu-Varsakelis (2006). We will first reduce the infinity of possible communication policies down to a set of sequences which guarantee the existence of an accompanying optimal LQG controller, and are easy to generate. That set will contain sequences which, for the purposes of LQG control, we will consider equally satisfactory, and which preserve the stabilisability and detectability of the underlying LTI plant which gives rise to (6).

Definition 3: The system (6) is controllable on $[k_0, k_f]$ if, given any x_0, there exists a control signal $\bar{u}(\cdot)$ that steers (6) from $x(k_0) = x_0$ to the origin at time k_f. We say that (6) is l-step controllable, or simply controllable if there exists a positive integer l such that (6) is controllable on $[k, k+l]$ for any k.

Definition 4: The system (6) is observable on $[k_0, k_f]$ if, given its input on $[k_0, k_f]$, any initial condition at k_0 can be uniquely determined by the output response $\bar{y}(k)$ for $k \in [k_0, k_f]$. We say that (6) is l-step observable, or simply observable if there exists a positive integer l such that (6) is observable on $[k, k+l]$ for any k.

Definition 5: The system (6) is reconstructible on $[k_0, k_f]$ if, given its input on $[k_0, k_f]$, $x(k_f)$ can be determined by the output response $y(k)$ for $k \in [k_0, k_f]$. We say that (6) is l-step reconstructible (or 'reconstructible') if there exists a positive integer l such that (6) is reconstructible on $[k, k+l]$, for any k.

It will be convenient to initially assume that the matrix A in (6) is invertible, so that controllability and reachability of (6) are equivalent, as are observability and reconstructability. The case where A may be singular will be treated later in this section. The following theorem and the sequence selection algorithm contained in its proof are obtained by modifying a similar result from Zhang and Hristu-Varsakelis (2006) to account for the matrix representation of the communication sequences, μ_ρ and μ_σ.

Theorem 1 (Zhang and Hristu-Varsakelis 2006): *Let the pair (A, B) be controllable, where B is $n \times m$, and A is invertible. For any integer $1 \le w_\rho < m$, there exist integers $l, N > 0$ and an N-periodic[1] m-to-w_ρ communication sequence $\rho(\cdot)$ such that the extended plant (6) is controllable on $[k, k+l]$ for all k, and thus controllable.*

Proof: Let

$$R \triangleq [A^{N-1}B\mu_\rho^T(0), A^{N-2}B\mu_\rho^T(1), \ldots, B\mu_\rho^T(N-1)]. \tag{8}$$

The system (6) is controllable on $[0, N]$ iff $\text{rank}(R) = n$. Notice that, at each step k, the input communication sequence $\mu_\rho^T(k)$ has the effect of 'selecting' w_ρ columns from the m columns of the term $A^{N-k-1}B$ on the RHS of (8). Also notice that for all $i = 0,1,\ldots$ the matrices $D_i = [A^{ni+n-1}B,\ A^{ni+n-2}B, \ldots, A^{ni}B]$, contain n linearly independent columns, because A is invertible and (A, B) is controllable. Then, in the worst case $w_\rho = 1$, the required sequence $\rho(\cdot)$ can be generated using the following algorithm:

(1) Let $D_i = [A^{ni+n-1}B,\ A^{ni+n-2}B, \ldots, A^{ni}B]$, where $i = 1, 2, \ldots$.
(2) Let $F_i = [f_i^0, \ldots, f_i^{n-1}]$ be a matrix formed by any n linearly independent columns from D_i.
(3) Let $F = F_0$.

(4) Replace f_0^1 in F by a column from F_1 while maintaining $\operatorname{rank}(F)=n$. Such a replacement can always be found because $\operatorname{rank}(F_1)=n$.
(5) For $i=2,\ldots,n-1$, replace f_0^i in F by a column from F_i while keeping the rank of F fixed. The resulting matrix F has one column from each D_i $(i=0,\ldots,n-1)$, and has rank n.

This algorithm ensures that it is possible to select n linearly independent columns as long as one can select one column from each D_i. For the less restrictive case $w_\rho > 1$, the communication sequence $\rho(\cdot)$ selects $n\cdot w_\rho$ columns from each D_i on the RHS of (8) (via the term $\mu_\rho^T(\cdot)$). Therefore, one can always produce (this time replacing w_ρ columns in F at each step) a sequence that selects n independent columns from the RHS of (8) in at most $k_f\lceil n/w_\rho\rceil\cdot n$ steps.

The algorithm described above yields a communication sequence $\rho(k)$, for $k=0,\ldots,N-1$, such that (6) is controllable on $[0,N]$, for some $N<n^2$. Now, extend $\rho(k)$ for $k\geq N$ by setting $\rho(k+N)=\rho(k)$. Because A is invertible, the N-periodic sequence $\mu_\rho(k)$ will select n independent columns from

$$[A^{jN+N-1}B\mu_\rho^T(0),\ldots,A^{jN}B\mu_\rho^T(jN+N-1)]$$

in every interval $[jN,(j+1)N-1]$, $j=0,1,2\ldots$ Let $l\geq 2N-1$. For all $k\geq 0$, there exists $j\geq 0$ such that $[jN,(j+1)N-1]\subset[k,k+l]$. Therefore, the periodic sequence $\mu_\rho(\cdot)$ will select n independent columns on $[k,k+l]$, for all k, and the extended plant is controllable under $\rho(\cdot)$. □

We note that the communication sequences produced using Theorem 1 are not optimised for period length, meaning that our algorithm does not attempt to identify the shortest-period controllability-preserving sequence. In practice, however, the algorithm produces sequences whose period is usually far shorter than the upper bound k_f. By switching from column manipulations to row manipulations, the duality of controllability and observability yields the following result whose proof is similar to that of Theorem 1.

Theorem 2: *Let the pair (A,C) be observable, where C is $p\times n$ and A is invertible. For any integer $1\leq w_\sigma<p$, there exist integers $l,N>0$ and an N-periodic p-to-w_σ communication sequence $\sigma(\cdot)$ such that the system (6) is observable on $[k,k+l]$ for all k, and thus observable.*

3.1 *NCS with stabilisable and detectable plants*

Because our objective is LQG control, preserving the plant's controllability and observability is more than is required. In fact, it will be sufficient to guarantee the weaker properties of stabilisability and detectability in the NCS. In the following, we describe how the results of § 3 can be used to do just that.

Definition 6: A discrete-time linear system is called stabilisable if its uncontrollable subsystem is stable; it is called detectable if its unreconstructible subsystem is stable.

Theorem 3: *Suppose the pair (A,B) is stabilisable, where B is $n\times m$, and A invertible. For any integer $1\leq w_\rho<m$, there exist integers $l,N>0$ and an N-periodic m-to-w_ρ communication sequence $\rho(\cdot)$ such that the NCS (6) is stabilisable.*

Proof: Suppose that (A,B) is stabilisable. Then, there exists an invertible matrix, Λ, which transforms the pair (A,B) into its Kalman canonical form:

$$\Lambda^{-1}A\Lambda=\begin{bmatrix}A_c & A_c'\\ 0 & A_{\bar{c}}\end{bmatrix},\quad \Lambda^{-1}B=\begin{bmatrix}B_c\\ 0\end{bmatrix},\tag{9}$$

where (A_c,B_c) and $(A_{\bar{c}},0)$ correspond to the controllable and uncontrollable subsystems of the pair (A,B), respectively. Define the new state variable

$$z\triangleq\Lambda^{-1}x=\begin{bmatrix}z_c\\ z_{\bar{c}}\end{bmatrix},$$

so that the dynamics of the extended plant (6) can be rewritten as

$$\begin{bmatrix}z_c(k+1)\\ z_{\bar{c}}(k+1)\end{bmatrix}=\begin{bmatrix}A_c & A_c'\\ 0 & A_{\bar{c}}\end{bmatrix}\begin{bmatrix}z_c(k)\\ z_{\bar{c}}(k)\end{bmatrix}+\begin{bmatrix}\bar{B}_c(k)\\ 0\end{bmatrix}\bar{u}(k),\tag{10}$$

where we have defined $\bar{B}_c(k)\triangleq B_c(k)\mu_\rho^T(k)$ for convenience. The uncontrollable subsystem $A_{\bar{c}}$ is stable because the pair (A,B) was assumed to be stabilisable. Note that (A_c,B_c) is controllable, and that A_c has non-zero eigenvalues because A was assumed to be invertible. Then, Theorem 1 implies that there exists a periodic communication sequence $\rho(k)$ such that the pair $(A_c,\bar{B}_c(\cdot))$ is controllable as well. Because the stabilisability of the extended plant (6) is invariant under change of coordinates, we conclude that the same sequence, $\rho(k)$, must preserve the stabilisability of the overall NCS, $(A,B\mu_\rho^T(k))$. □

Theorem 4 is dual theorem for detectability.

Theorem 4: *Let the pair (A,C) be detectable, where C is $p\times n$, and A invertible. For any integer $1\leq w_\sigma<p$,* there exist integers *$l,N>0$ and an N-periodic p-to-w_σ communication sequence $\sigma(\cdot)$ such that the NCS (6) is detectable.*

3.2 *The case of non-invertible A*

If the matrix A_c in (9) (corresponding to the plant's controllable subsystem) is not invertible, then the sequence selection algorithm of Theorem 1 cannot be used directly to obtain the results of the previous Section. It is still possible, however, to construct sequences that preserve the stabilisability and detectability of the NCS by considering only the subspace that corresponds to the non-zero eigenvalues of A (or the non-zero eigenvalues of A_c in the proof of Theorem 3).

Theorem 5: *Let the pair* (A, B) *be stabilisable, where* B *is* $n \times m$. *For any integer* $1 \le w_\rho < m$, *there exist integers* $l, N > 0$ *and an N-periodic m-to-*w_ρ *communication sequence* $\rho(\cdot)$ *such that the extended plant is stabilisable.*

Proof: The case where A is invertible was addressed in Theorem 3. Suppose then that A has q $(1 \le q < n)$ zero eigenvalues; there exists an invertible matrix, Φ, such that

$$\Phi^{-1} A \Phi = \begin{bmatrix} A_1 & 0 \\ 0 & A_0 \end{bmatrix}, \quad \Phi^{-1} B = \begin{bmatrix} B_1 \\ B_0 \end{bmatrix},$$

where A_1 is $(n-q) \times (n-q)$ invertible, A_0 is $q \times q$ with only zero eigenvalues, B_1 is $(n-q) \times m$, and B_0 is $q \times m$. Define the new state variable

$$z \triangleq \Phi^{-1} x = \begin{bmatrix} z_1 \\ z_0 \end{bmatrix},$$

where $z_1 \in \mathbb{R}^{n-q}$, $z_0 \in \mathbb{R}^q$. Then, the dynamics of the extended plant (6) can be rewritten as

$$\begin{bmatrix} z_1(k+1) \\ z_0(k+1) \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_0 \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_0(k) \end{bmatrix} + \begin{bmatrix} \bar{B}_1(k) \\ \bar{B}_0(k) \end{bmatrix} \bar{u}(k), \tag{11}$$

where $\bar{B}_0 = B_0 \mu_\rho^T(k)$ and $\bar{B}_1(k) = B_1 \mu_\rho^T(k)$. In (11), the extended plant (6) has been decomposed into two uncoupled subsystems. The controllability (also reachability, stabilisability) and reconstructibility (also observability, detectability) of a linear system do not change under similarity transformations. Therefore, studying the controllability and reconstructibility of the extended plant (6) is equivalent to studying the same properties in (11).

We distinguish between two cases, depending on the controllability of the pair (A, B). First, if (A, B) is controllable then (A_1, B_1) must be controllable as well. Because A_1 is invertible, Theorem 1 implies that there exists an integer $k_1 > 0$ and a communication sequence $\rho(k)$, for $k \in [0, k_1]$, such that the pair $(A_1, \bar{B}_1(\cdot))$ is controllable on $[0, k_1]$. Hence, for any initial condition, there exists a control $\bar{u}(k)$, for $k \in [0, k_1]$, that steers z_1 to zero at $k = k_1$. Apply the pair $\rho(k)$, $\bar{u}(k)$ during $[0,\ k_1]$; for $k > k_1$, set $\bar{u}(k) = 0$, and let the communication sequence $\rho(k)$ be arbitrary. Then, there must exist $k_f > k_1$, such that $z_0(k_f) = 0$ because all of A_0's eigenvalues are zero. Also, z_1 must have remained at zero during $[k_1 + 1,\ k_f]$, because $\bar{u}(k) = 0$ during that time. We have thus constructed a control sequence $\bar{u}(k)$ and a communication sequence $\rho(k)$ on $[0,\ k_f]$ such that, starting from any initial conditions, the state z is driven to the origin at time k_f. This is equivalent to stating that (11) is controllable (also, stabilisable) on $[0,\ k_f]$ under the communication sequence $\rho(k)$. Now, we can extend $\rho(\cdot)$ to be periodic, using the same argument as in the proof of Theorem 1, and conclude that the extended plant will be stabilisable under the same sequence $\rho(k)$ by the invariance of that property under change of coordinates.

Finally, if (A, B) is stabilisable but not controllable, we first transform the pair to its Kalman canonical form (as in Theorem 3) and then repeat the previous arguments for the controllable part (A_c, B_c) of (A, B). □

The duality of controllability and reconstructibility gives the following.

Theorem 6: *Let the pair* (A, C) *be detectable, where* C *is* $p \times n$. *For any integer* $1 \le w_\sigma < p$, *there exist integers* $l, N > 0$ *and an N-periodic p-to-*w_σ *communication sequence* $\sigma(\cdot)$ *such that the NCS* (6) *is detectable.*

4. LQG problem formulation and solution

We now return to NCSs in which the plant is stochastic. Modifying the discussion of § 2 to account for the presence of sensor and measurement noise in (1), results in (6) being replaced by the stochastic extended plant

$$\left.\begin{aligned} x(k+1) &= Ax(k) + \bar{B}(k)\bar{u}(k) + v(k) \\ \bar{y}(k) &= \bar{C}(k)x(k) + \bar{w}(k), \end{aligned}\right\} \tag{12}$$

where for convenience we have defined $\bar{B}(k) \triangleq B\mu_\rho(k)^T$, $\bar{C}(k) \triangleq \mu_\sigma(k)C$, and $\bar{w}(k) \triangleq \mu_\sigma(k)w(k)$. Having relaxed the requirement for optimal communication, as per the discussion in § 3, we now restate the LQG problem.

Problem 2: Given a pair of input and output sequences $(\rho(\cdot), \sigma(\cdot))$ which preserve the stabilisability and detectability of (1) in the NCS (12), design an optimal controller that minimises

$$J = \mathcal{E}\left\{\sum_{k=0}^{N_f} x^T(k)Qx(k) + \bar{u}^T(k)\bar{u}(k)\right\}. \tag{13}$$

Because $\sigma(\cdot)$ is determined off-line and $\bar{w}(k)$ is a sub-vector of $w(k)$, the $\bar{w}(k)$ and $v(k)$ are independent random variables, with $\bar{w}(k) \sim N(0, I_{w_\sigma \times w_\sigma})$. Thus, Problem 2 is a standard LQG problem for the time-varying stochastic extended plant (12). It is well known (see, for example, Bertsekas (2000)) that the solution is comprised of (i) a Kalman filter that produces the optimal state estimate $\hat{x}(k)$ given the output $\bar{y}(0), \ldots, \bar{y}(k)$, and (ii) an LQ optimal feedback controller, $\bar{u}^*(k) = -L(k)\hat{x}(k)$, where the gain, $L(k)$, is obtained by solving a deterministic LQ problem with perfect state information. The separation principle ensures that the two subproblems can be solved independently.

In the following, we review the analytical expressions for the Kalman filter and optimal controller; the proofs of these results can be found in most stochastic control texts e.g., Bertsekas (2000).

4.1 *Kalman filtering*

The optimal estimator for the extended plant (12) is the discrete time Kalman filter, described in two recursive steps, with $\hat{x}(0) = x_0$, $\Sigma(0) = \Sigma_0$. The first is a time update of the conditional mean $\hat{x}(k^-)$ of the state $x(k)$ prior to the measurement of $\bar{y}(k)$,

$$\hat{x}(k^-) = A\hat{x}(k-1) + \bar{B}(k-1)\bar{u}(k-1) \tag{14}$$

$$P(k) = A\Sigma(k-1)A^T + G, \tag{15}$$

where G is the variance of the noise term $v(k)$ in (6), and $P(k)$ is the variance of the prediction error. The second step is a measurement update

$$H(k) = P(k)\bar{C}^T(k)\left(\bar{C}(k)P(k)\bar{C}^T(k) + I\right)^{-1} \tag{16}$$

$$\hat{x}(k) = \hat{x}(k^-) + H(k)\left(\bar{y}(k) - \bar{C}(k)\hat{x}(k^-)\right) \tag{17}$$

$$\Sigma(k) = \left(I - H(k)\bar{C}(k)\right)P(k), \tag{18}$$

where $\hat{x}(k) \triangleq \mathcal{E}\{x(k)|\bar{y}(0)\cdots\bar{y}(k)\}$ is the state estimate, and $\Sigma(k)$ is the covariance of the estimation error.

The sequence $P(k+1)$ satisfies the time-varying discrete time Riccati equation

$$\begin{aligned} P(k+1) = {} & AP(k)A^T + G \\ & - AP(k)\bar{C}^T(k)\left[I + \bar{C}(k)P(k)\bar{C}^T(k)\right]^{-1} \\ & \cdot \bar{C}(k)P(k)A^T. \end{aligned} \tag{19}$$

The 'one-step prediction' error, $e(k) = x(k) - \hat{x}(k^-)$, satisfies

$$\mathcal{E}e(k+1) = (A - \Gamma(k)\bar{C}(k))\mathcal{E}e(k), \tag{20}$$

where $\Gamma(k) \triangleq AH(k)$ is the Kalman gain.

4.2 *LQ optimal control*

The optimal control law for the LQG problem is obtained by solving a standard LQ problem for the deterministic version of (12), assuming that state feedback is available. The optimal controller is $\bar{u}^*(k) = -L(k)x(k)$, where $L(k)$ satisfies

$$L(k) = \left(\bar{B}^T K(k+1)\bar{B}(k) + I\right)^{-1}\bar{B}(k)^T K(k+1)A, \tag{21}$$

and the symmetric positive semidefinite $K(k)$ satisfy the backwards Riccati equation

$$\begin{aligned} K(N_f) &= Q, \\ K(k) &= A^T K(k+1)A + Q \\ &\quad - A^T K(k+1)\bar{B}(k)\left(\bar{B}^T(k)K(k+1)\bar{B}(k) + I\right)^{-1} \\ &\quad \cdot \bar{B}^T(k)K(k+1)A. \end{aligned} \tag{22}$$

The closed loop dynamics for the deterministic state equation under optimal control are

$$x(k+1) = (A - \bar{B}(k)L(k))x(k). \tag{23}$$

4.3 *Periodic Riccati equations*

From § 3, it follows that if the plant (1) is stabilisable and detectable, it is always possible to design periodic communication sequences $\sigma(\cdot)$ and $\rho(\cdot)$ that preserve those properties in the extended plant (12). Under periodic communication, $\bar{B}(k)$, $\bar{C}(k)$ are periodic as well. Therefore, the Riccati equations (19), (22) associated with the LQG problem both become Discrete-time Periodic Riccati Equations (DPREs). DPREs have been studied extensively, e.g., in Bittanti et al. (1988) and references therein; we go on to review three of the relevant results from that work.

Definition 7 (Bittanti et al. 1998)**:** A Discrete-time Periodic Riccati Equation (DPRE) is a difference equation of the form

$$\begin{aligned} \mathcal{P}(k+1) = {} & \mathcal{A}(k)\mathcal{P}(k)\mathcal{A}^T(k) + \mathcal{B}(k)\mathcal{B}(k)^T \\ & - \mathcal{A}(k)\mathcal{P}(k)\mathcal{C}^T(k)[I + \mathcal{C}(k)\mathcal{P}(k)\mathcal{C}^T(k)]^{-1} \\ & \cdot \mathcal{C}(k)\mathcal{P}(k)\mathcal{A}^T(k), \end{aligned} \tag{24}$$

where $\mathcal{A}(k): \mathbb{Z} \mapsto \mathbb{R}^{n \times n}$, $\mathcal{B}(k): \mathbb{Z} \mapsto \mathbb{R}^{n \times m}$, $\mathcal{C}(k): \mathbb{Z} \mapsto \mathbb{R}^{p \times n}$ are N-periodic.

Theorem 7 (Bittanti et al. 1998, Theorem 5)**:** *Consider the Kalman gain* $\mathcal{K}(k)$

$$\mathcal{K}(k) = \mathcal{A}(k)\mathcal{P}(k)\mathcal{C}^T(k)(\mathcal{C}(k)\mathcal{P}(k)\mathcal{C}^T(k) + I)^{-1}$$

associated with any symmetric positive semidefinite solution $\mathcal{P}(\cdot)$ *of* (24). *If* $(\mathcal{A}(\cdot), \mathcal{B}(\cdot))$ *is stabilisable and* $(\mathcal{A}(\cdot), \mathcal{C}(\cdot))$ *detectable, then the corresponding closed-loop matrix* $\hat{\mathcal{A}}(\cdot) = \mathcal{A}(\cdot) - \mathcal{K}(\cdot)\mathcal{C}(\cdot)$ *is exponentially stable.*

Theorem 8 (Bittanti et al. 1998, Theorem 6): *There exists a unique Symmetric Periodic Positive Semidefinite (SPPS) solution $\bar{\mathcal{P}}(\cdot)$ of the DPRE* (24) *and $\bar{\mathcal{A}}(\cdot) = \mathcal{A}(\cdot) - \bar{\mathcal{K}}(\cdot)\mathcal{C}(\cdot)$ is asymptotically stable iff $(\mathcal{A}(\cdot), \mathcal{B}(\cdot))$ is stabilisable and $(\mathcal{A}(\cdot), \mathcal{C}(\cdot))$ is detectable, where $\bar{\mathcal{K}}(\cdot)$ is the Kalman gain associated with $\bar{\mathcal{P}}(\cdot)$.*

Theorem 8 gives a necessary and sufficient condition for the existence and uniqueness of an SPPS solution as well a stability condition for the closed-loop system. The next result guarantees the asymptotic convergence of the DPRE to the unique SPPS solution, as $k \to \infty$.

Theorem 9 (Bittanti et al. 1998, Theorem 7): *Suppose that $(\mathcal{A}(\cdot), \mathcal{B}(\cdot))$ is stabilisable and $(\mathcal{A}(\cdot), \mathcal{C}(\cdot))$ detectable. Then, every symmetric and positive semidefinite solution of the DPRE* (24) *converges to the unique SPPS solution as $k \to \infty$.*

4.4 *Convergence of the LQG optimal controller*

Theorem 6, combined with the results of § 4.3 implies the following:

Theorem 10: *Suppose that in the NCS* (12)

(1) *The output communication sequence $\sigma(\cdot)$ is chosen to be N-periodic and such that the detectability of the the plant* (1) *is preserved in the extended plant.*
(2) *The pair (A, g) is stabilisable, where $gg^T = G$.*

Then, starting from any positive definite initial conditions, (19) *converges to a unique N-periodic solution, $\bar{\Sigma}(k)$, as $k \to \infty$. Moreover, the error dynamics* (20) *are exponentially stable.*

The theorem follows by making the identifications $\mathcal{A} \to A$, $\mathcal{B} \to g$, $\mathcal{C} \to C$, $\mathcal{P} \to P$, and $\mathcal{K} \to AH$ in Theorems 7–9. The next result is obtained in a similar fashion.

Theorem 11: *Suppose that in the NCS* (12),

(1) *The input communication sequence $\rho(\cdot)$ is N-periodic and such that the stabilisability of the the plant* (1) *is preserved in the extended plant.*
(2) *The pair (A, q^T) is detectable, where $qq^T = Q$.*

Then, starting from any positive definite initial conditions, the Riccati equation associated with the LQ problem (22) *converges to a unique N-periodic solution $\bar{K}(k)$ as $k \to \infty$. Moreover, the closed loop dynamics* (23) *are exponentially stable.*

5. When transmission delays are present

We now consider NCSs where in addition to the medium access constraints discussed up to now, controller-plant communication is also subject to known transmission delays. In the remainder of this section we outline a delay-compensation method which is inspired by Luck and Ray (1990), and can be combined with the LQG controller and communication sequences developed previously. A starting point for the treatment of random delays in this context could be Lincoln and Bernhardsson (2000b), Wang et al. (2003) and Chan and Ozguner (1995).

5.1 *Continuous-time plants: reduction to the discrete-time case*

If the underlying plant evolves in discrete time, then one can simply amend the NCS model (12) to include any (integer) delays. We will take the plant-to-controller and controller-to-plant delays to be $\Delta_1, \Delta_2 > 0$, respectively:

$$\left.\begin{aligned} x(k+1) &= Ax(k) + \bar{B}(k-\Delta_2)\bar{u}(k-\Delta_2) + v(k) \\ \bar{y}(k) &= \bar{C}(k-\Delta_1)x(k-\Delta_1) + \bar{w}(k-\Delta_1), \end{aligned}\right\} \tag{25}$$

If, on the other hand, the plant evolves in continuous time, we will require that its output is sampled periodically, with period $T > 0$, and will take the triple (A, B, C) to be the parameters of the resulting sampled-data system. Upon receipt of a set of output data, $\bar{y}$ (Figure 2), the controller computes and transmits a control $\bar{u}$, which will reach the plant τ_{cp} time units later. We will assume that the controller computes inputs instantaneously; if that is not the case, the analysis that follows must be modified by increasing τ_{cp} to include the time needed to compute an input. Unless $\tau_{cp} + \tau_{pc}$ is a multiple of the sampling period T, an input will arrive at the plant 'between' samplings (see Figure 3). It will be convenient to

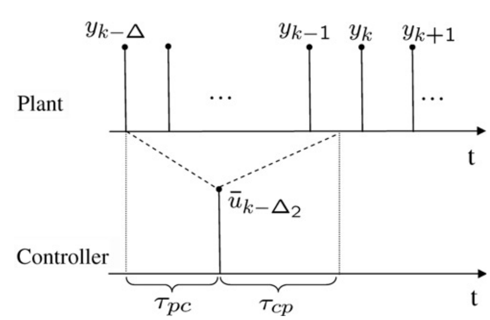

Figure 3. NCS with continuous-time plant and delays. Plant outputs are sampled periodically and arrive at the controller τ_{cp} time units after transmission. The controller computes and transmits $\bar{u}(k-\Delta_2)$, which arrives at the plant with a delay of τ_{cp}, and is applied at the next sampling instant, Δ_2 periods following transmission. Here, $\Delta_2 = \lceil \tau_{\mathrm{cp}}/T \rceil$ and $\Delta = \lceil (\tau_{cp} + \tau_{pc})/T \rceil$.

arrange matters so that input data arriving during $(kT, kT+T]$ will be 'buffered' at the plant's input stage and will be applied to the actuators at the next sampling instant, $kT+T$. This assumption can easily be lifted, but we will make use of it here because it allows us to simplify matters by reducing the case of a continuous-time plant to the discrete-time model (25). An alternative is to require that incoming inputs are applied immediately, without waiting for the next sampling instant. Taking into account the intersample behaviour of the plant when control inputs are applied upon arrival is straightforward and requires adjusting the predictor equations that are given next, in order to correctly estimate the plant state at the time of arrival of a control input. The modifications are in the spirit of what is described in Zhang et al. (2001) and will be omitted.

For an NCS with continuous-time plant, we will label quantities as follows. Let $u(k)$ denote the input that arrived at the plant some time in $(kT, kT+T]$, while $x(k)$ and $y(k)$ will be the samples of the state and output, respectively, at $t=kT$. Likewise, $\bar{y}(k)$ will be the output data that arrived at the controller during $(kT, kT+T]$. Inputs produced by the controller will be indexed by the time interval in which they were computed, i.e., $\bar{u}(k)$ will be the input produced by the controller during $(kT, kT+T]$. Because plant inputs are buffered and outputs are sampled periodically, the controller will generate precisely one such input per sampling interval. The output received by the controller will then be related to the 'true' output by

$$\bar{y}(k) = \mu_\sigma(k-\Delta_1)y(k-\Delta_1),$$

where $\Delta_1 = \lceil \tau_{pc}/T \rceil$, and μ_σ is as per Def. 2, while the input received by the plant is related to that which the controller generates by

$$u(k) = \mu_\rho^T(k-\Delta_2)\bar{u}(k-\Delta_2),$$

where $\Delta_2 = \lceil \tau_{pc}/T \rceil$. The resulting stochastic extended plant has precisely the same expression as (25), subject to the interpretation of the various signals as indicated above.

5.2 *Delay compensation*

For each time k, we will construct a family of predictors, whose states $\hat{x}(k,i)$ are the estimates of the plant's 'true' state $x(k-\Delta_1+i)$. These predictors are to be updated by the controller each time it receives new data from the plant.

- We begin by modifying the Kalman filter equations, using data which was transmitted to the controller Δ_1 time steps (resp. sampling periods) in the past:

$$\begin{aligned}\hat{x}(k^-,1) &= A\hat{x}(k-1,1) + \bar{B}(k-\Delta-1)\bar{u}(k-\Delta-1),\\ \hat{x}(0^-,1) &= x_0\\ \hat{x}(k,1) &= \hat{x}(k^-,1) + H(k)\big(\bar{y}(k) - \bar{C}(k-\Delta_1)\hat{x}(k^-,1)\big),\end{aligned} \tag{26}$$

  where $\Delta = \Delta_1 + \Delta_2$ and the gain $H(k)$ is as in §4.1.
- The estimate $\hat{x}(k^-,1)$ is then propagated forward in time, in order to estimate the state at the time the controller's currently-generated output $\bar{u}(k)$ is to reach the plant:

$$\hat{x}(k^-, j+1) = A\hat{x}(k^-, j) + \bar{B}(k-\Delta-1+j) \cdot \bar{u}(k-\Delta-1+j), \quad j = 1,\ldots,\Delta \tag{27}$$

- The estimate $\hat{x}(k^-, \Delta+1)$ of the future state $x(k+\Delta_2)$ is then used in place of state feedback:

$$\bar{u}(k) = L(k)\hat{x}(k^-, \Delta+1). \tag{28}$$

When applying the delay compensator (26)–(28), we will choose to forgo the measurement update in (26) for the first Δ_1 time steps, and instead rely only on prediction until the controller receives the first set of output data.

5.3 *Closed loop dynamics with medium access constraints and delays*

Theorem 12: *For the NCS* (6) *and predictor* (27), *define the estimation error to be*

$$e(k) \triangleq x(k-\Delta_1) - \hat{x}(k^-,1). \tag{29}$$

Under the feedback controller (28), *the closed loop dynamics of* (25) *satisfy*

$$\begin{bmatrix} \mathcal{E}x(k+1) \\ \mathcal{E}e(k-\Delta_2+2) \end{bmatrix} = \mathcal{A}(k) \begin{bmatrix} \mathcal{E}x(k) \\ \mathcal{E}e(k-\Delta_2+1) \end{bmatrix}, \tag{30}$$

where

$$\mathcal{A}(k) \triangleq \left[\begin{array}{c|c} A + \bar{B}(k-\Delta_2)L(k-\Delta_2) & V(k) \\ \hline 0 & A - \Gamma(k-\Delta_2+1)\bar{C}(k-\Delta+1) \end{array}\right], \tag{31}$$

and $V(k)$ is a matrix that depends on the (finite) time histories of $\bar{B}$, $\bar{C}$, H and L.

Before we prove Theorem 12, we will need the following intermediate result whose proof can be found in the Appendix.

Lemma 1: *For all $k=1,2,\ldots$, and $r=1,2,\ldots$,*

$$\begin{aligned}\mathcal{E}\hat{x}(k^-, r+1) &= \mathcal{E}\hat{x}(k+r^-, 1) - A\sum_{i=0}^{r-1} A^i H(k+r-i-1)\\ &\quad\cdot \bar{C}(k+r-i-1-\Delta_1)\mathcal{E}e(k+r-i-1). \qquad (32)\end{aligned}$$

Proof of Theorem 12: From the definition of the estimation error (29) and the observer dynamics (26), it is easy to verify that $\mathcal{E}e(k+1) = (A + AH(k)\bar{C}(k-\Delta_1))\mathcal{E}e(k)$ or, using $\Gamma(k) \triangleq AH(k)$,

$$\mathcal{E}e(k+1) = (A - \Gamma(k)\bar{C}(k-\Delta_1))\mathcal{E}e(k), \qquad (33)$$

which establishes (after time-shifting by $\Delta_2 - 1$ steps) the lower-right block of $\mathcal{A}(k)$ in (31).

Under the feedback law (28), the NCS evolves according to

$$\begin{aligned}x(k+1) &= Ax(k) + \bar{B}(k-\Delta_2)\bar{u}(k-\Delta_2) + v(k)\\ &= Ax(k) + \bar{B}(k-\Delta_2)L(k-\Delta_2)\\ &\quad\cdot \hat{x}(k-\Delta_2^-, \Delta+1) + v(k). \qquad (34)\end{aligned}$$

Using Lemma 1, the last equation yields

$$\begin{aligned}\mathcal{E}x(k+1) &= A\mathcal{E}x(k) + \bar{B}(k-\Delta_2)L(k-\Delta_2)\\ &\quad\cdot\Bigg(\hat{x}(k+\Delta_1^-, 1) - A\sum_{i=0}^{\Delta-2} A^i H(k+\Delta-2-i)\\ &\quad\cdot \bar{C}(k+\Delta-2-i-\Delta_1)\mathcal{E}e(k+\Delta-2-i)\Bigg) \qquad (35)\end{aligned}$$

By adding and subtracting the term $\bar{B}(k-\Delta_2)\times L(k-\Delta_2)\mathcal{E}x(k)$ from the right-hand side of the last equation, we obtain

$$\begin{aligned}\mathcal{E}x(k+1) &= (A + \bar{B}(k-\Delta_2)L(k-\Delta_2))\mathcal{E}x(k)\\ &\quad - \bar{B}(k-\Delta_2)L(k-\Delta_2)\mathcal{E}e(k+\Delta_1)\\ &\quad + \text{ terms involving } \mathcal{E}e(k+\Delta-2), \ldots, \mathcal{E}e(k+1),\\ &\quad \mathcal{E}e(k). \qquad (36)\end{aligned}$$

By expressing $\mathcal{E}e(k+\Delta-i-2)$ in terms of $\mathcal{E}e(k-\Delta_2+1)$ by means of (33), we have

$$\begin{aligned}\mathcal{E}x(k+1) &= (A + \bar{B}(k-\Delta_2)L(k-\Delta_2))\mathcal{E}x(k)\\ &\quad - \bar{B}(k-\Delta_2)L(k-\Delta_2)\mathcal{E}e(k+\Delta_1)\\ &\quad - \sum_{i=0}^{\Delta-2}\Bigg(A^i H(k+\Delta_1-1-i)\bar{C}(k-i-1)\\ &\quad \cdot \prod_{j=k-\Delta_2+2}^{k+\Delta_1} W(j)\mathcal{E}e(k-\Delta_2+1)\Bigg), \qquad (37)\end{aligned}$$

where $W(j) \triangleq A - \Gamma(j)\bar{C}(j-\Delta_1)$.

To obtain the top row of (30)–(31) we again express $\mathcal{E}e(k+\Delta_1)$ in terms of $\mathcal{E}e(k-\Delta_2+1)$ in (37), and define $V(k)$ to be the sum of all factors that multiply $\mathcal{E}e(k-\Delta_2+1)$ in the resulting equation for $\mathcal{E}x(k+1)$. □

Equation (31) suggests that in order to apply the LQG controller of § 4 to the case where delays are imposed, the observer gain sequence, $H(\cdot)$, should be time-shifted by Δ_1 steps, while the controller gains $L(\cdot)$ should be unchanged.

6. A numerical example

To illustrate our approach, we simulated the 3-input, 3-output, 6th order unstable LTI plant with parameters

$$A = \begin{bmatrix} 1.1 & 0 & 0 & 0 & 0 & 0\\ -1.5 & 0 & -0.75 & -1.5 & 0.75 & -0.75\\ -1.1 & 0 & 0 & -1.1 & 0 & 0\\ 0 & 0 & 0 & 1.1 & 0 & 0\\ 1.1 & 0.75 & 0 & 1.1 & 0 & -0.75\\ -0.75 & 0 & -0.75 & -0.75 & 0 & -0.75 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 1\\ 1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & -1\\ 1 & 0 & 1\\ 0 & -1 & 0 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 1 & -1\\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}.$$

The disturbance terms in (1) were $v(\cdot) \sim N(0, 0.35 I_{6\times 6})$ and $w(\cdot) \sim N(0, I_{3\times 3})$. We formulated an infinite-horizon version of the LQG problem (Problem 2 in §4), with $Q = 4I_{6\times 6}$ and initial conditions $x(0) = [1,\ 50,\ 7,\ 6,\ 1,\ 2]^T$, $\hat{x}(0) = 0$, and $\Sigma(0) = 0.2 \cdot I_{6\times 6}$. The choice of an infinite horizon meant that the steady-state SPPS solutions to the Riccati equations (19) and (25) were required in order to determine the controller and observer gains, and that we only needed to store as many gains as the communication sequence period at run time. The plant was controlled through a shared communication medium which had only one input and one output channels, i.e., $w_\rho = w_\sigma = 1$.

The plant's Kalman decomposition reveals that the plant is indeed stabilisable and detectable, with a 1-dimensional stable controllable/unobservable subsystem, a 4-dimensional unstable controllable/observable subsystem, and a 1-dimensional stable uncontrollable/observable subsystem. The proof of Theorems 3 and 5 indicates that in order to obtain a communication sequence that maintains stabilisability (detectability) in the presence of communication constraints, it is sufficient to identify the sequence that does the job for the controllable (reconstructible) part of the plant. Using the algorithm contained in the proof of Theorem 1, we found that the period-3 sequences

$$\{\sigma(0), \sigma(1), \ldots, \} = \{[1,0,0]^T, [0,1,0]^T, [0,1,0]^T, \ldots, \},$$

$$\{\rho(0), \rho(1), \ldots, \} = \{[1,0,0]^T, [1,0,0]^T, [0,0,1]^T, \ldots, \},$$

preserve the controllability and reconstructibility of the 4-dimensional unstable subsystem, and thus guarantee that the NCS is stabilisable and detectable, so that we can construct the stabilising LQG controller of §4. It is interesting to note that in this case, the 'round-robin' policies $\sigma(k) = \rho(k) = \{[1,\ 0,\ 0]^T, [0,\ 1,\ 0]^T, [0,\ 0,\ 1]^T, \ldots\}$ do not result in a stabilisable or detectable system.

In the first of two simulations, we took the communication medium to be delay-free ($\tau_{pc} = \tau_{cp} = 0$). The solution of the periodic Riccati equation (19), associated with the Kalman filter error covariance, converged to a 3-periodic SPPS solution $\bar{P}(\cdot)$ in approximately 20 steps. The same was the case for the solution of the periodic backwards Riccati equation (22), associated with the LQ optimal gain. The evolutions of $tr(P(k))$ and $tr(K(k))$ are shown in Figure 4. Using the solutions for $P(\cdot)$ and $K(\cdot)$, the Kalman filter and the LQ optimal feedback gain were computed from the formulae in §4.1, 4.2. The observer and controller gains were

$$\bar{H}(3i) = \begin{bmatrix} 0.3862 \\ 0.1117 \\ 0.2541 \\ -0.5684 \\ -0.1748 \\ 0.0404 \end{bmatrix}, \quad \bar{H}(3i+1) = \begin{bmatrix} 0.5213 \\ -0.0253 \\ -0.6189 \\ 0.0976 \\ 0.6204 \\ -0.0347 \end{bmatrix},$$

$$\bar{H}(3i+2) = \begin{bmatrix} 0.4236 \\ -0.0334 \\ -0.5382 \\ 0.1146 \\ 0.5232 \\ -0.0540 \end{bmatrix},$$

$$\bar{L}(3i) = \Big[0.3107 \quad -0.0410 \quad -0.2250, \quad -0.0532 \quad 0.2146 \quad -0.184\Big],$$

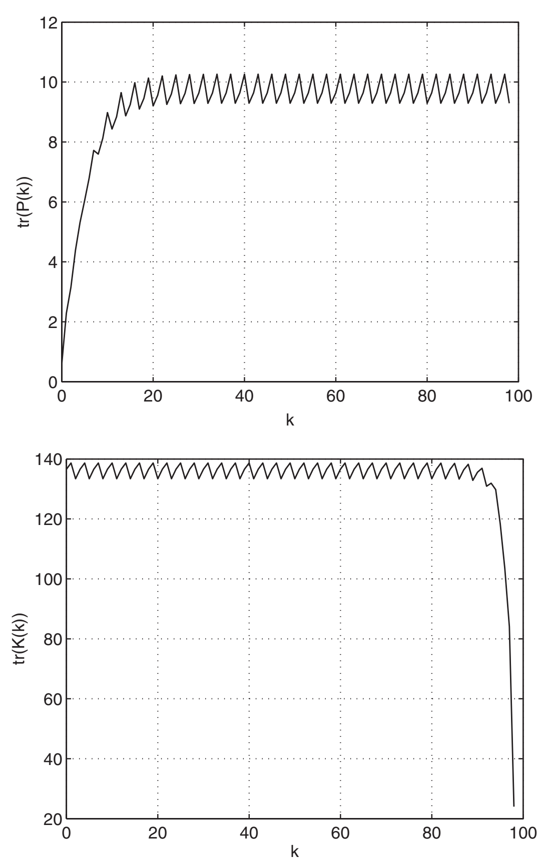

Figure 4. Evolution of $tr(P(k))$ and $tr(K(k))$. The $P(k)$ satisfy the Riccati equation (19)), while $K(k)$ satisfy the backwards Riccati equation (25).

$$\bar{L}(3i+1) = \begin{bmatrix} 0.7138 & 0.0854 & -0.0564 \\ & 0.6997 & 0.1226 & -0.1418 \end{bmatrix},$$

$$\bar{L}(3i+2) = \begin{bmatrix} 0.2707 & 0.082 & 0.003 & -0.6824 \\ & -0.0588 & -0.079 \end{bmatrix}, \qquad (38)$$

where $i = 0, 1, 2, \ldots$.

The state evolution of the closed-loop system under optimal control is shown in Figure 5. For the sample path shown there, all states entered a small 'band' around the origin after about 20 steps. The evolution of the Kalman filter's one-step prediction error, $e(k)$, is shown in Figure 6.

Next, we simulated the same NCS, starting from the same initial conditions, this time with a controller-to-plant delay of $\Delta_2 = 3$ time steps and a plant-to-controller delay of $\Delta_1 = 2$ time steps, in addition to the access constraints $w_\rho = w_\sigma = 1$. The observer/predictor from § 5.3 were used in order to compensate for the effects of transmission delays, and the gain sequences from the delay-free case were applied. The plant's state evolution is shown in Figure 7. The predictor error $e_p(k) \triangleq x(k) - \hat{x}(k, \Delta_1 + 1)$ is shown in Figure 8.

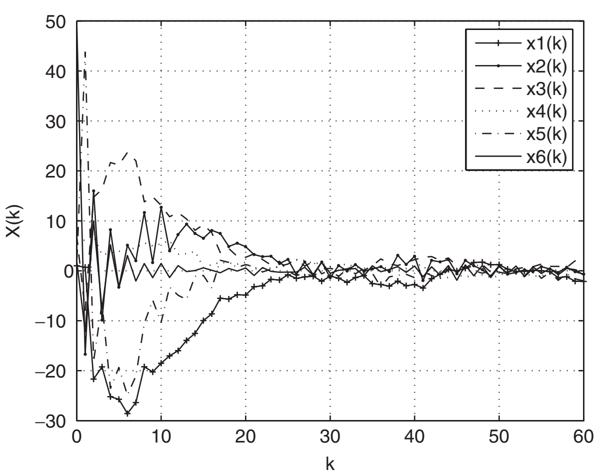

Figure 5. State evolution of the closed-loop NCS under the LQG controller. Delay-free case.

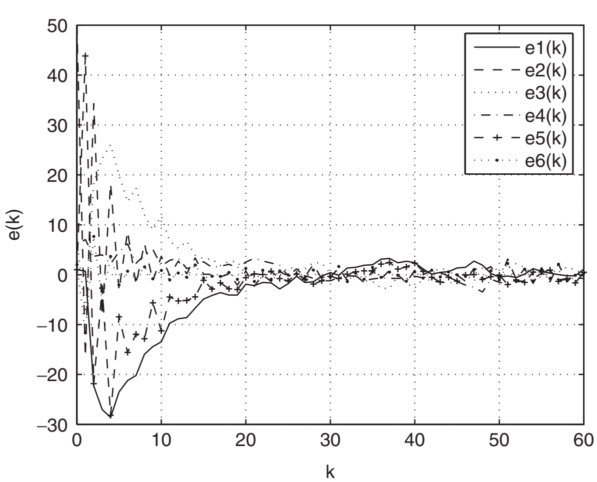

Figure 6. Evolution of Kalman filter's one-step prediction error $e(k) = x(k) - \hat{x}(k^-)$. Delay-free case.

7. Conclusions

We presented an LQG design method for NCSs which are subject to medium access constraints and known delays. Our approach forgoes the use of ZOH elements in the loop; instead, the controller and plant 'ignore' sensors and actuators which are not granted medium access. The selection of communication sequences is decoupled from the choice of controller by relaxing the requirement for jointly optimal control and communication, thereby simplifying the identification of useful communication patterns and allowing us to bring existing tools to bear. Specifically, we showed that it is always possible to design periodic communication sequences that preserve the detectability and stabilisability of the underlying plant in the presence of communication constraints. Having doing so, Kalman filtering and LQ optimal control of an NCS can be formulated as a standard LQG problem for an equivalent periodic time-varying system. Our choice of communication sequences ensures that the Riccati equations associated with the Kalman filter and the LQ optimal gain both converge (for infinite-horizon problems) to periodic solutions regardless of initial conditions. Thus, the associated stabilising optimal LQG controller can be easily implemented.

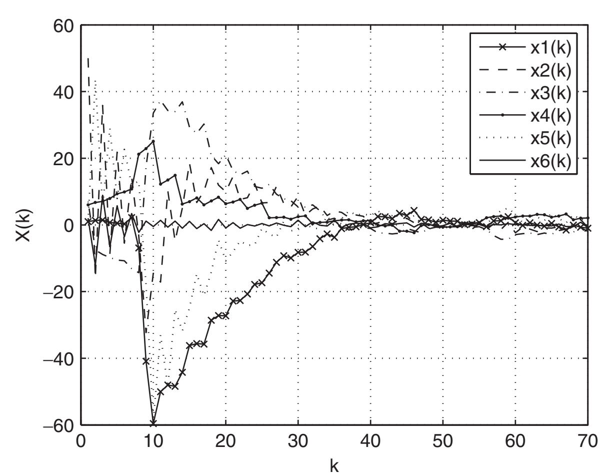

Figure 7. State evolution of the closed-loop NCS under the LQG controller, with transmission delays $\Delta_1 = 2$, $\Delta_2 = 3$.

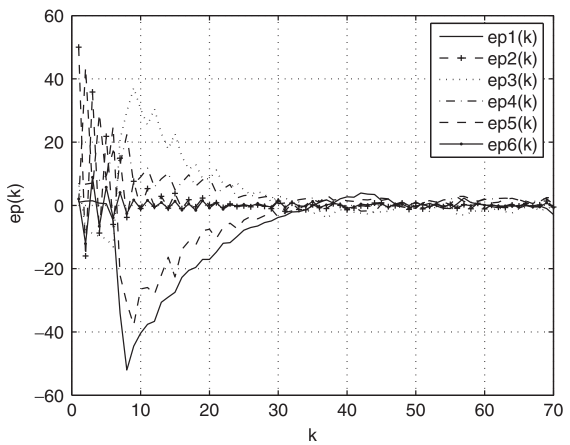

Figure 8. Predictor error evolution for the closed-loop NCS, with transmission delays $\Delta_1 = 2$, $\Delta_2 = 3$. The prediction error $e_p(k) = x(k) - \hat{x}(k, \Delta_1 + 1)$ is plotted only subsequently to the first reception of output data by the controller ($k > \Delta_1$).

The proposed approach to LQG control of NCS is comprised of three components:

- A pair of periodic communication sequences which are designed off-line, independently of the controller. Periodic communication sequences can be easily implemented via MAC-level network protocols such as polling, token passing, or time division multiple access (TDMA).
- A periodic time-varying linear controller whose parameters can be computed off-line.
- A delay compensator which is implemented at the controller.

Our method does not address the joint optimisation of the controller and the communication sequences, a problem which currently appears to be intractable. Using the notion of a communication sequence as a basic modelling tool, one can consider extensions of this work to NCSs which are subject to 'dropped' data packets and random delays, in addition to the constraints discussed here.

Note

1. A discrete-time communication sequence $\sigma(\cdot)$ will be called N-periodic if $\sigma(k) = \sigma(k+N)$ for all k.

Appendix

Proof of Lemma 1: The statement is proved by induction. First, we verify that the statement holds for $r = 1$. From the observer equations (26) we have

$$\begin{aligned}&\mathcal{E}\hat{x}(k+1^-,1)\\&\quad = A\mathcal{E}\hat{x}(k^-,1) + \bar{B}(k-\Delta)\bar{u}(k-\Delta) + AH(k)\bar{C}(k-\Delta_1)\\&\qquad \cdot \underbrace{(x(k-\Delta_1) - \hat{x}(k^-,1))}_{\mathcal{E}e(k)},\end{aligned} \tag{39}$$

while the predictor (27) implies that

$$\mathcal{E}\hat{x}(k^-,2) = A\mathcal{E}\hat{x}(k^-,1) + \bar{B}(k-\Delta)\bar{u}(k-\Delta).$$

Combining the last two equations, we obtain

$$\mathcal{E}\hat{x}(k^-,2) = \mathcal{E}\hat{x}(k+1^-,1) - AH(k)\bar{C}(k-\Delta_1)\mathcal{E}e(k).$$

One can also verify (again using (26) and (27)) that the statement holds for $r = 2$. Assuming now that (32) holds (for an index of $r+1$), we show that it also holds for index $r+2$. From (26) we have that

$$\begin{aligned}&\hat{x}(k+r+1^-,1)\\&\quad = A\hat{x}(k+r,1) + \bar{B}(k+r-\Delta)\bar{u}(k+r-\Delta)\\&\quad = A\mathcal{E}\hat{x}(k+r^-,1) + \bar{B}(k+r-\Delta)\bar{u}(k+r-\Delta)\\&\qquad + AH(k+r)\Big(\bar{C}(k+r\Delta_1)x(k+r-\Delta_1)\\&\qquad + \bar{w}(k+r-\Delta_1) - \bar{C}(k+r-\Delta_1)\hat{x}(k+r^-,1)\Big)\\&\quad \Rightarrow \mathcal{E}\hat{x}(k+r+1^-,1)\\&\quad = A\mathcal{E}\hat{x}(k+r^-,1) + \bar{B}(k+r-\Delta)\bar{u}(k+r-\Delta)\\&\qquad + AH(k+r)\bar{C}(k+r-\Delta_1)\mathcal{E}e(k+r),\end{aligned} \tag{40}$$

while (27) implies that

$$\hat{x}(k^-,r+2) = A\hat{x}(k^-,r+1) + \bar{B}(k-\Delta+r)\bar{u}(k-\Delta+r). \tag{41}$$

Using our hypothesis that (32) holds, the last equation yields

$$\begin{aligned}&\mathcal{E}\hat{x}(k^-,r+2)\\&\quad = A\mathcal{E}\hat{x}(k+r^-,1) - A\sum_{i=0}^{r-1} A^i H(k+r-i-1)\\&\qquad \cdot \bar{C}(k+r-i-1-\Delta_1)\mathcal{E}e(k+r-i-1)\\&\qquad + \bar{B}(k-\Delta+r)\bar{u}(k-\Delta+r).\end{aligned} \tag{42}$$

By comparing (40) and (42), we obtain the desired result:

$$\begin{aligned}&\mathcal{E}\hat{x}(k^-,r+2)\\&\quad = \mathcal{E}\hat{x}(k+r+1,1) - AH(k+r)\bar{C}(k+r-\Delta1)\mathcal{E}e(k+r)\\&\qquad - A\sum_{i=0}^{r-1}\Big(A^{i+1}H(k+r-1-i)\bar{C}(k+r-1-i-\Delta_1)\\&\qquad \cdot \mathcal{E}e(k+r-1-i)\Big)\\&\quad = \mathcal{E}\hat{x}(k+r+1,1) - A\sum_{i=0}^{r}\Big(A^i H(k+r-i)\\&\qquad \bar{C}k+r-i-\Delta_1)\mathcal{E}e(k+r-i)\Big).\end{aligned} \tag{43}$$

□

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